**Feasibility detection.** Suppose that you want to solve a linear program but you don't have a starting initial basic feasible solution—perhaps the all 0 vector is not feasible. Design a related linear program whose solution will be a basic feasible solution to the original linear program (assuming the original linear program has a basic feasible solution).

**Detecting unboundedness.** Describe how to modify the simplex algorithm to detect an unbounded linear program—a linear program in which there is a feasible solution that makes the objective function arbitrarily large.

**Birkhoff-von Neumann theorem.** Consider the polyhedron *P* defined by∑*i*​*xij*​=1,∑*j*​*xij*​=1, and *xij*​≥0. Prove that all extreme points of *P* have integer coordinates (0 or 1).